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Composite Materials

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*NanoCem-møde at BTG*DTU, Copenhagen, 29/april, 2003*

Volume V, Y'oungs modulus E, Subscript P/S indicate phase P/S

$$c = \frac{V_P}{V_P + V_S} \quad \text{Volume concentration of phase } P \tag{1}$$

$$e = \frac{E}{E_S} \ \text{Composite stiffness:} \quad n = \frac{E_P}{E_S} \ \text{Stiffness ratio}$$

Layered geometry (anisotropic)

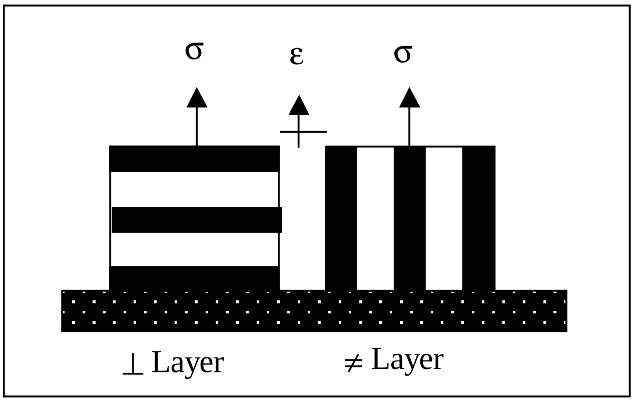

Figure 1. *Extreme anisotropic composites*

$$e = \begin{cases} \dfrac{n}{n + c(1 - n)} \ \perp \text{Layer} \\ \\ 1 + c(n - 1)\_\ \text{Layer} \end{cases} \tag{2}$$

Isotropic geometry with Composite Spheres Assemblage (Isotropic)

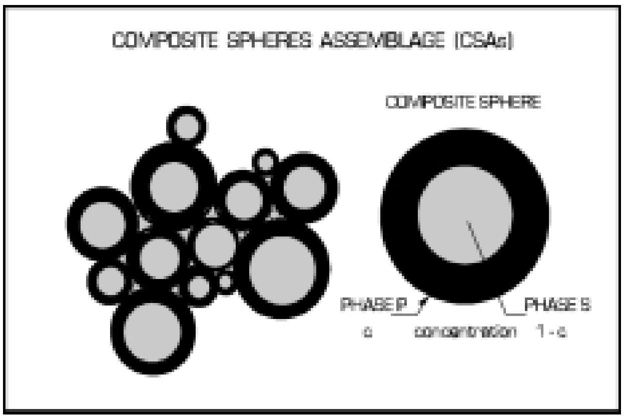

Figure 2. *Composite Spheres Assemblage with phase S particles of concentration 1-c*

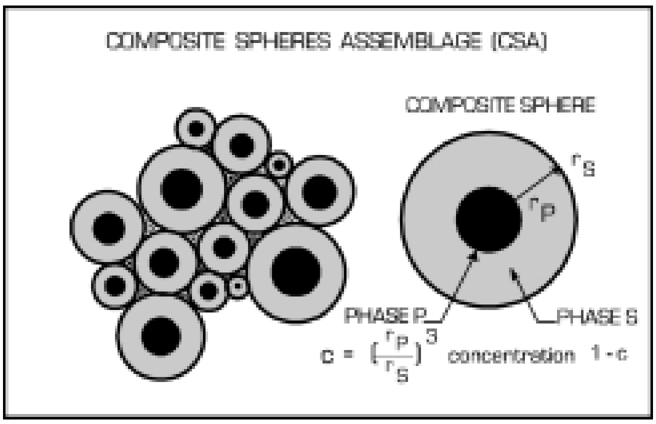

Figure 3. *Composite Spheres Assemblage with phase P particles of concentration c*

$$e = \begin{cases} \dfrac{n+1+c(n-1)}{n+1-c(n-1)} & (CSA_P) \\ n\dfrac{2+c(n-1)}{2n-c(n-1)} & (CSA_S) \end{cases} \tag{3}$$

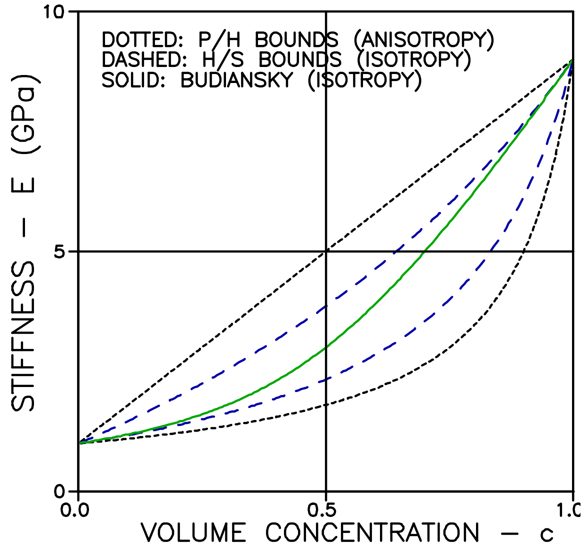

Figure 4 *P/H bounds, H/S bounds, and Budiansky's expression.*

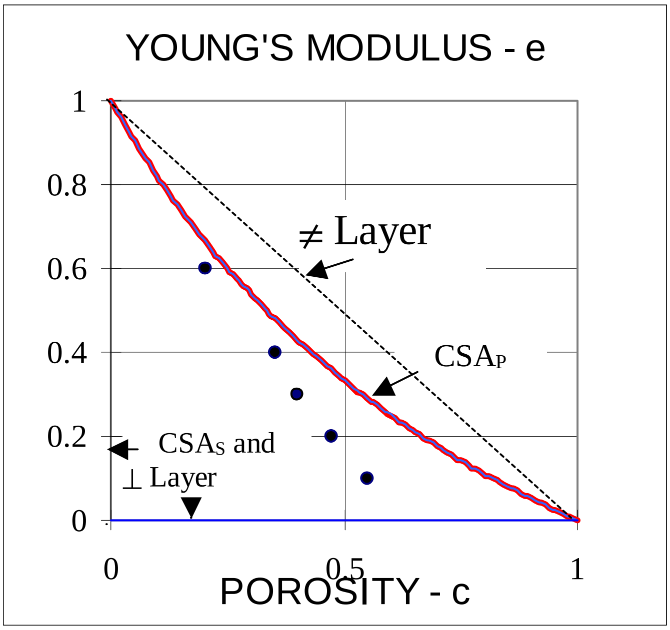

Figure 5. *Porous materials. Dots are stiffness experimentally deter-mined*

General geometry

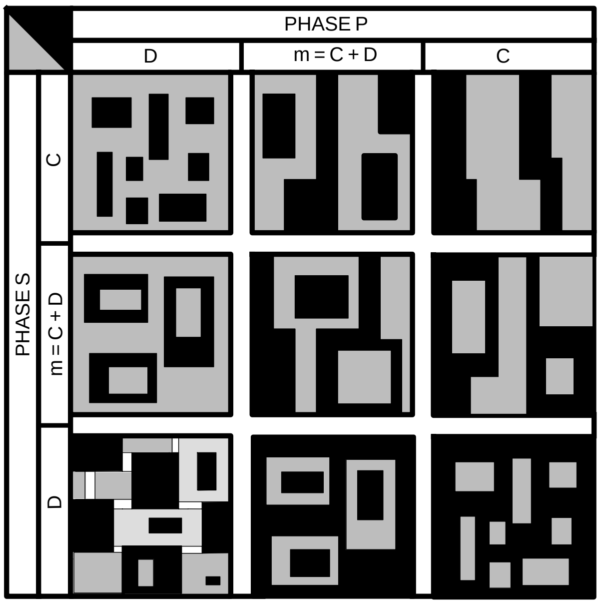

Figure 6. *Phase geometries in two-phase materials. C, D and m (= C + D) denote continuous geometry, discrete geometry, and mixed geometry respectively*

Stiffness of composites with isotropic geometry

$$e = \frac{n + \theta[1 + c(n-1)]}{n + \theta - c(n-1)} \quad \textit{with geo-function}$$

$$\theta = \frac{1}{2}\left[\mu_P + n\,\mu_S + \sqrt{(\mu_P + n\,\mu_S)^2 + 4n(1 - \mu_P - \mu_S)}\right]; \; n = \frac{E_P}{E_S} \tag{4}$$

The Young's modulus, expressed by the former expression (with the geometry function θ) is global, meaning that the solution is invariable with respect to composite geometry. The geo-function is restricted with respect to stiffness ratio as shown in Figure 7.

At any concentration, c, specific composite geometries are quantified by shape functions μ_P and μ_S. Examples are presented in Figure 7 with associated percolation graphs. Detailed shape functions for a so-called DC-CD composite are demonstrated in Figure 8.

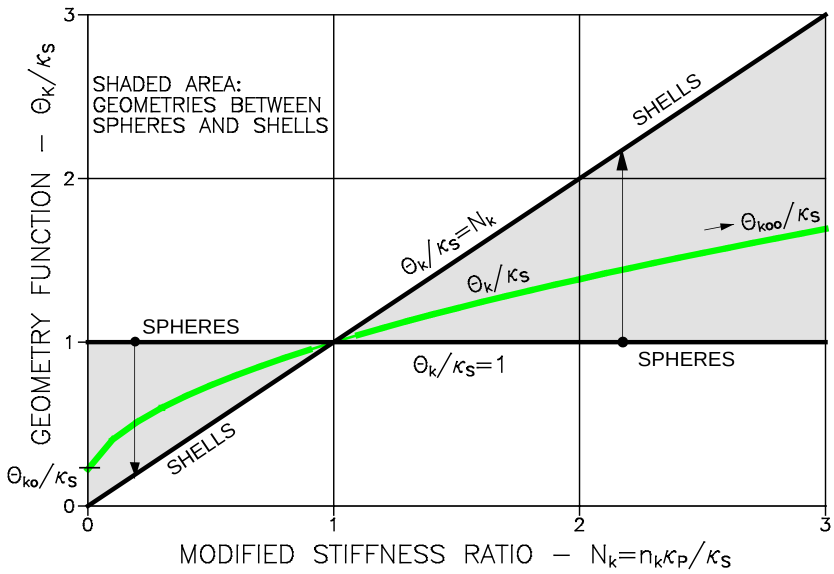

Figure 7. *Influence of phase P geometry on geometry-function,* θ

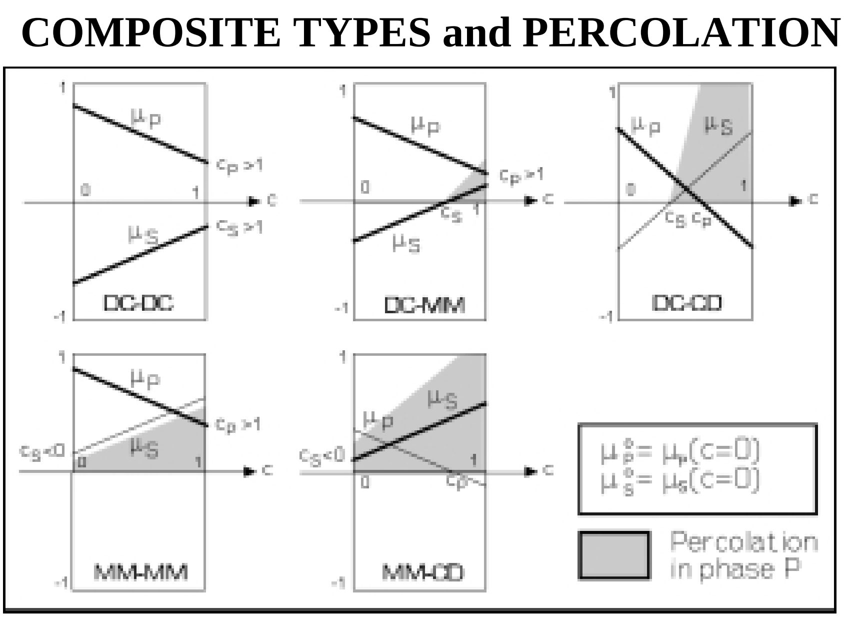

Figure 8. *Composite types versus critical concentrations. Former and latter two letters denote composite geometry at c = 0 and at c = 1 respectively.*

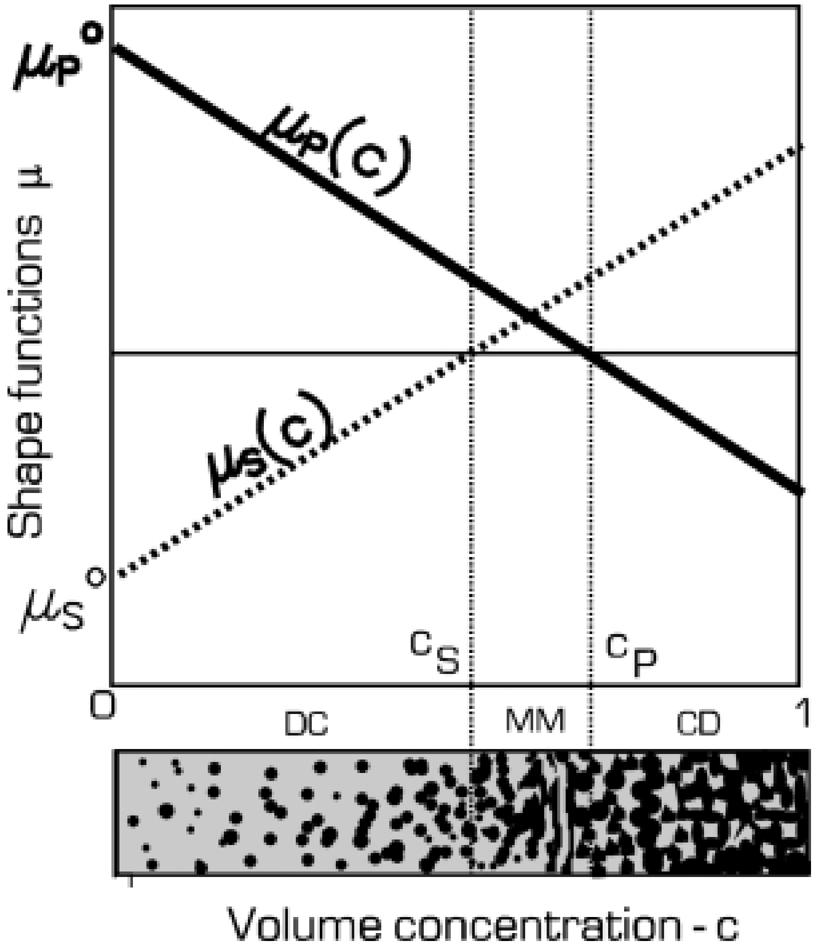

Figure 9. *Geometrical significance of shape functions. (μ_P, μ_S) = (+,-) means discrete P in continuous S. (μ_P,μ_S) = (+,+) means mixed P in mixed S. (μ_P,μ_S) = (-,+) means continuous P with discrete S. Black and white signatures denote phase P and phase S respectively.*

Spin-off results

Important spin-off results from developing Equation 4 are that stress and strain in the constituent phases are simultaneously determined

- and that composite conductivities can be expressed in a very similar way as stiffness is determined (using the same shape functions).

Conductivity (Q)

$$q = \frac{Q}{Q_S} = \frac{n_Q + \theta_Q [1 + c(n_Q - 1)]}{n_Q + \theta_Q - c(n_Q - 1)} \quad \text{with geo - function}$$

$$\theta_Q = \mu_P + n_Q \mu_S + \sqrt{(\mu_P + n_Q \mu_S)^2 + 4 n_Q (1 - \mu_P - \mu_S)} \;\; ; \;\; n_Q = \frac{Q_P}{Q_S} \tag{5}$$

Also important is that stress and strain as well as flow of matters and potentials in composite components can be predicted.

SUMMARY OF RESULTS

Stiffness and eigenstrain/stress

Stiffness

$$e = \frac{E}{E_S} = \frac{n + \theta[1 + c(n - 1)]}{n + \theta - c(n - 1)} \tag{6}$$

Stress due to external mechanical load

$$\frac{\sigma_P}{\sigma} = \frac{n(1 + \theta)}{n + \theta[1 + c(n - 1)]} \;\; ; \;\; \frac{\sigma_S}{\sigma} = \frac{n + \theta}{n + \theta[1 + c(n - 1)]} \tag{7}$$

Eigenstrain – linear (λ)

$$\lambda = \lambda_S + \Delta\lambda \frac{1/e - 1}{1/n - 1} \;\; ; \;\; (\Delta\lambda = \lambda_P - \lambda_S) \tag{8}$$

Eigenstress – hydrostatic (ρ)

$$\rho_P = -\frac{5}{3} E_S \Delta\lambda \frac{c(1/n-1)-(1/e-1)}{c(1/n-1)^2} \quad ; \quad \rho_S = -\frac{c}{1-c}\rho_P \tag{9}$$

Conductivity (Q)

$$q = \frac{Q}{Q_S} = \frac{n_Q + \theta_Q [1 + c(n_Q - 1)]}{n_Q + \theta_Q - c(n_Q - 1)} \quad ; \quad n_Q = \frac{Q_P}{Q_S} \tag{10}$$

Examples

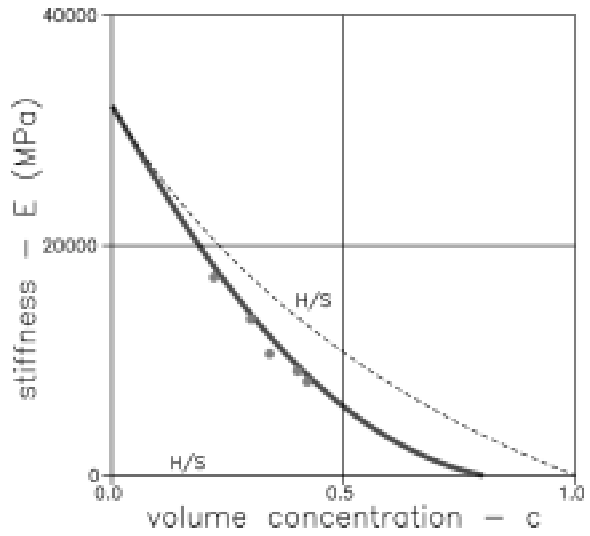

***Figure 10.** Example 2: Stiffness of cement paste as related to capillary porosity.*

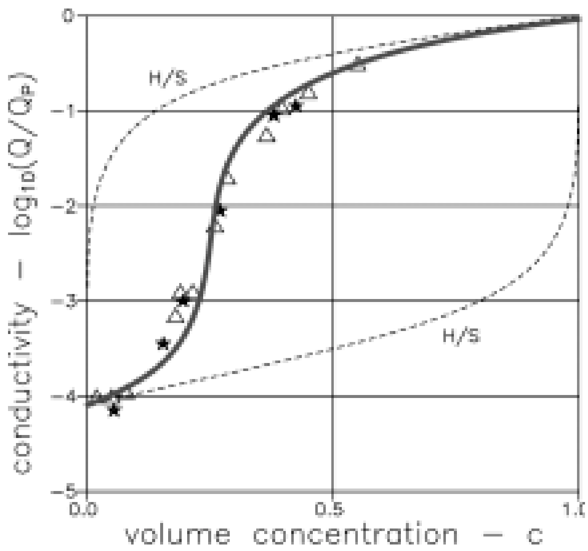

***Figure 11.** Example 2: Chloride diffusivity of cement paste as related to capillary porosity.*

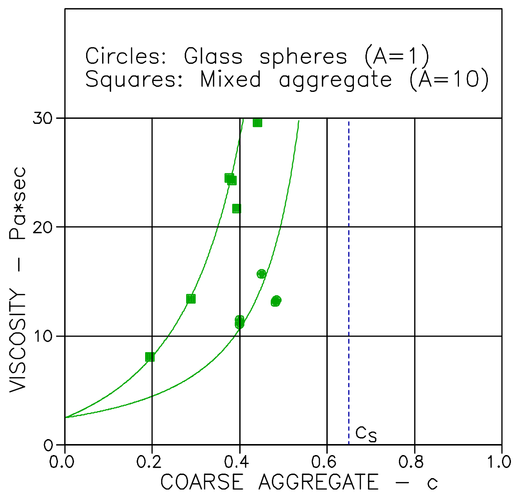

***Figure 12.** Viscosity of concrete as related to volume fraction of coarse aggregates (18). Solid lines are predicted with M_V = 1. Mortar viscosity is η_S = 2.5 Pa*sec. c_S = 0.65.*

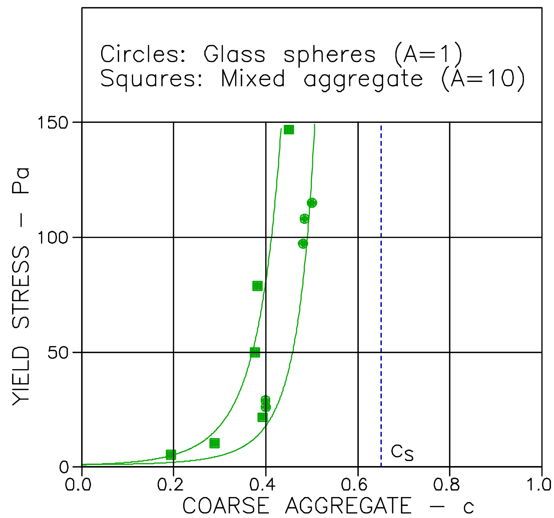

***Figure 13.** Yield stress of concrete as related to volume fraction of coarse aggregates (18). Solid lInes are predicted with M_S = 3.5. Mortar yield stress is S_S = 1 Pa. c_S = 0.65.*

Advantages, using composite theory

- Composite property solutions (global) can be determined which are valid for any materials composition, meaning that number of calibrations to be made in practice is reduced considerably.
- The solutions obtained ‘cover’ both mechanical and physical properties.
- Further more, the solutions are, by analogies, also valid for viscoelastic composites – including composite liquids.

Obviously composite theory is the most important basis for developing the discipline of ‘materials design’: How can we construct materials with prescribed material properties.

Materials design

Present state

- We know, how global (geometrically independent) mechanical/physical material properties can be determined for composite materials only from knowing about

  Volumetric composition and phase properties of the constituent phases

- We know the concept of, how to convert these global properties to properties for composites with specific geometries – using a geometry function (θ)

$$\theta = \frac{1}{2}\left[\mu_P + n\mu_S + \sqrt{(\mu_P + n\mu_S)^2 + 4n(1-\mu_P-\mu_S)}\right] \text{ with shape functions } (\mu)$$

- These two observations form the theoretical basis of materials design: Shape functions (phase geometries) required to obtain a desired material property can be obtained by using the global property solution (or analogy expressions) with the geometry function just presented.

In practice, we now ‘only’ have to produce the composite geometry predicted.

Future

- Further theoretical research to refine the descriptions of shape- and geometry functions
- Develop efficient 'inverse' mathematical tools – to determine conditions (composite geometry) from known solutions (composite properties)
- Practical research on technologies, which can produce pre-described composite geometries

Further examples of composite analysis

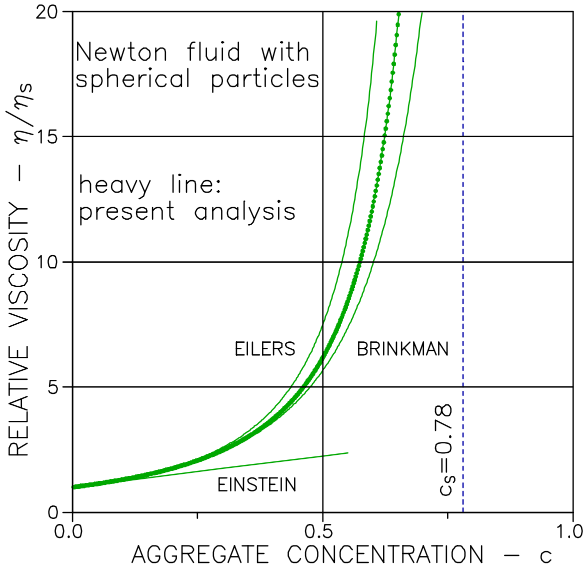

Figure 14. *Spherical particles (A = 1) in a viscous matrix. Present analysis and empirical descriptions by Eilers and Brinkman.*

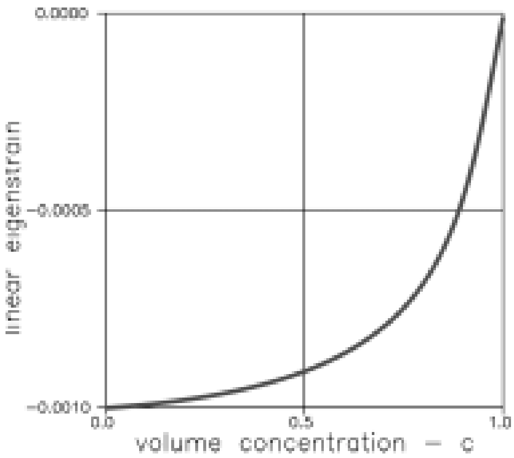

Figure 15. *Example 1: Linear composite eigenstrain (negative shrinkage).*

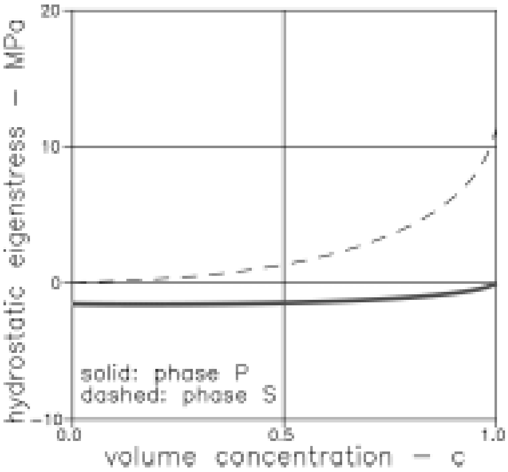

Figure 16. *Example 1: Hydrostatic stress caused by shrinkage of matrix* (S).

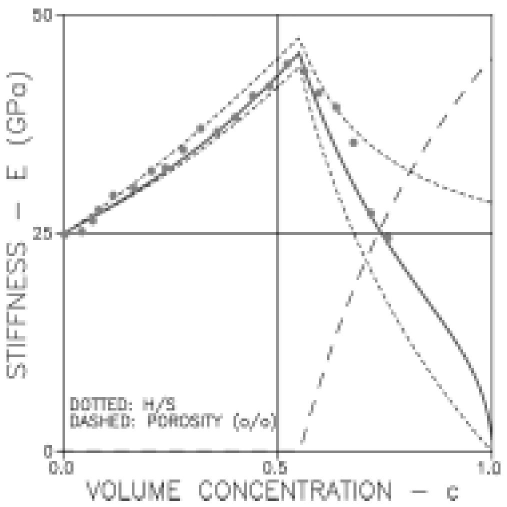

Figure 17. *Example 4: Stiffness of cement mortar with uni-sized quartz sand.*

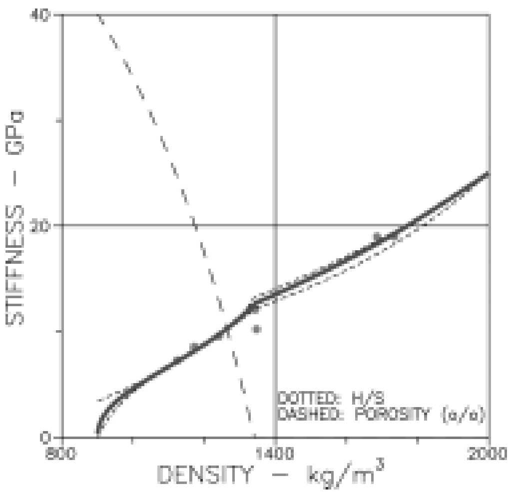

Figure 18. *Example 3: Stiffness of light clinker concrete. Non-flexible phase P.*

Materialenyt 1:2001, DSM (Danish Society for Materials Testing and Research)

Numerical analysis of composite materials

Lauge Fuglsang Nielsen

http://www.byg.dtu.dk/publicering/software_d.htm.

Skitse til foredrag
(!!skal ændres siden sidste anvendelse af overheads!!)

OVERHEAD 1

- Når vi taler om kompositmaterialer tror jeg at de fleste tænker på sammensatte materialer, der kan modelleres ved parallel- eller kuglemodeller.

- De résummeres i Figurerne 1, 2, og 3 med stivheder som vist i Ligningerne 2 og 3.

OVERHEAD 2

- Sætvist er modellerne hinandens ekstreme geometriske modsætninger.

I Figur 1 kan vi ikke modellere mere ’modsat’ end vist for anisotrope kompositmaterialer.

I Figur 2 kan vi ikke modellere mere ’modsat’ end vist for isotrope kompositmaterialer.

- Derfor beskriver de angivne stivhedsudtryk, sætvist, grænser for stivheden af anisotrope henholdsvis isotrope kompositmaterialer. Eksempler er vist i Figur 4.

- For et porøst materiale beskrives de normerede stivheder som vist i Figur 5, hvor typiske eksperimentelle værdier er indsat med sorte klatter.

- De viste grænser i Figur 5 er exacte. I forhold til ’naturen’ er der noget galt, som den hidtidige kompositteori har overset. Som nævnt er de viste eksperimentelle resultater meget typiske.

- Eneste mulighed er, at naturens kompositgeometrier ikke kan modelleres så simpelt som vist i Figurerne 1 og 2.

OVERHEADS 3 og 4

- Vi må opfinde en kompositteori, der kan tage hensyn til alle tænkelige geometrier som de er stiliserede i Figur 6. *FORKLAR !*

- Resultatet i Ligning 4 viser et eksempel på nyere forskning på dette felt. Det angivne stivhedsudtryk er globalt. Det vil sige, det gælder for alle isotrope kompositgeometrier. Hensyn til specifikke geometrier sker gennem så-kaldte formfunktioner (μ) i den så-kaldte geometrifunktion (θ).

- Geometri funktioner skal overholde grænser som vist i Figur 7

- Formfunktioner ser typisk ud som vist i Figur 8

OVERHEAD 5

Formfunktioner er relateret til percolation som vist i Figur 9.

Bemærk 'Spin-off' resultatet fra arbejdet med stivhedsanalyserne. Konduktivitets- og stivhedsanalyser sker på fuldstændig samme måde. Indflydelsen af kompositgeometrier er den samme.

Delkomponenternes spændinger og tøjninger bestemmes samtidigt med stivheder.

OVERHEAD 6

Her er et resumé af nogle væsentlige resultater fra den nye kompositteori

OVERHEAD 7

- Nogle eksempler på kompositeanalyser.

OVERHEADS 8 og 9

- Fordele ved at anvende kompositanalyser.
- Bemærk, at de også kan anvendes på viskoelastiske kompositmaterialer
- Kompositteorien anbefaler umiddelbart sig selv som det naturlige udgangspunkt for 'materialedesign'
- Materialedesign – perspektiver

OVERHEADS 10 og 11

Flere eksempler på compositanalyse